

CONTROL SYSTEMS

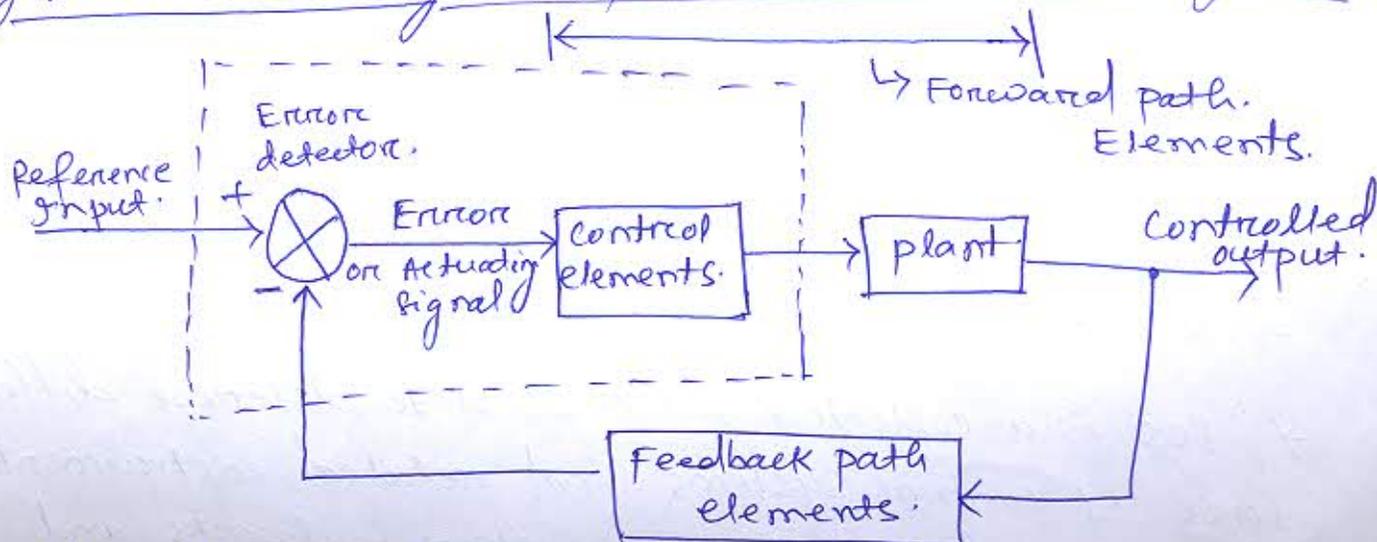
MODULE - I

Page ①

Class - 01. 20/08/20.

Control system \rightarrow The collection and combination of elements in a planned manner to getting a desired output is called control system.

General block diagram of an automatic control system \rightarrow



Advantages of Automatic Control System \rightarrow

- \rightarrow Cost of energy or power reduced.
- \rightarrow Cost of processing materials in industries reduces.
- \rightarrow quality of product improve.
- \rightarrow productivity increases.

Examples Refrigeration control, toilet tank, water level control and hot water heater control are the examples of automatic control devices which are commonly used for domestic purpose.

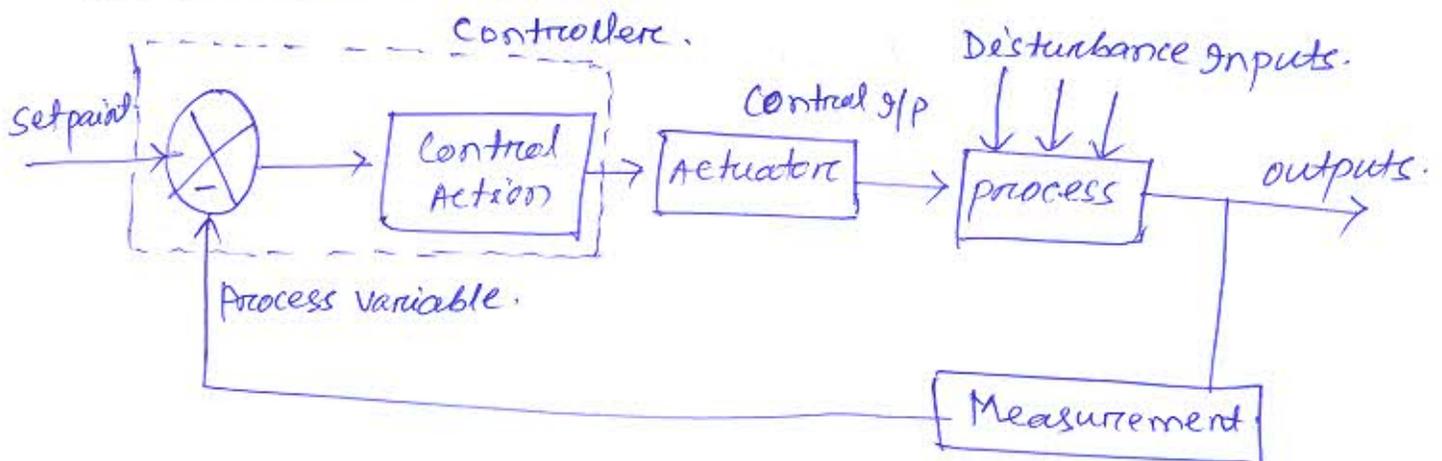
Industrial Control System :->

The following are ICS technologies —

- > Supervisory control and data acquisition (SCADA)
- > Distributed control systems (DCS)
- > Industrial Automation and Control systems (IACS)
- > Programmable Logic Controllers (PLCs)
- > Programmable Automation Controllers (PACs)
- > Human Machine Interface (HMI)
- > Remote Terminal Units (RTUs)

* ICS is a collective term used to describe different types of control systems and related instrumentation, which include the devices, systems, networks and controls used to operate and/or automate industrial processes.

* A DCS is also commonly used in industries such as manufacturing, electric power generation, chemical manufacturing, oil refineries, and water and wastewater treatment.



INTRODUCTION TO CONTROL SYSTEMS:ENGINEERING:

- Engineering combine the fields of Science & math to solve real world problem that improves the world around us
- Consider something used in every day life building, Roads, bridges, vehicles (car, bus, plane) Computer & other electronic device.
- Not a single one of them would exist without Engineering

SYSTEM:

- A set of detailed method, procedures & routines created to carry out a specific activity, perform a duty & solve a problem.

CONTROL:

- It means to regulate or to command a system so that we get a desired output.

CONTROL SYSTEM:

- Procedures, designed & established to check, record, regulate, supervise, authenticate & (if necessary) restrict, the access to an asset resource or system.

OR

- It is the collection & combination of elements in a planned manner whose each elements produce & effect to give the desired output.

CONTROL SYSTEM ENGINEERING:

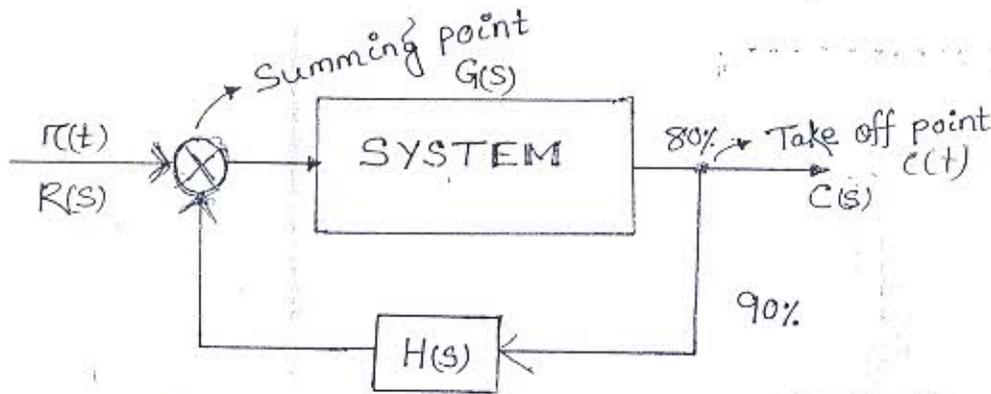
- It is a part of the Engg discipline that applies control theory to design system with desired behaviours.

PLANT:

- It is a part of the system which will control or regulated it is other wise known as process.

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FEED BACK PATH OR FEED BACK ELEMENT :

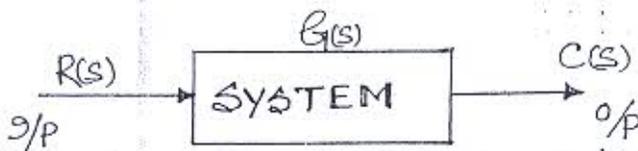


- It is the path or element by which we can compare the output of the system with the i/p for getting a desired o/p.
- Depending upon this feed back path control system is 2 type.

- i) Open Loop Control System.
- ii) Closed Loop Control System.

i) OPEN LOOP CONTROL SYSTEM :

- It is otherwise called non-feed back type control system.

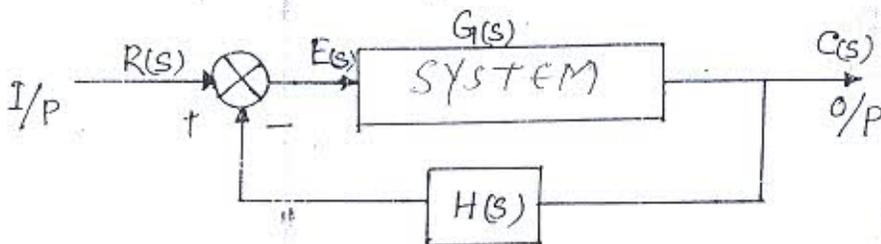


- In this type the o/p is independent of the i/p of the system.
- It is time based type control system.

- EX: (i) Washing M/C
(ii) Traffic Light System.

ii) CLOSED LOOP CONTROL SYSTEM :

- It is otherwise called feed back type control system.
- The o/p of this type of system is depended on the i/p.
- Here a feed back path is present by which we can compare the o/p with the input.



5

$R(s)$ = Laplace Transformation of i/p signal.

$C(s)$ = Laplace Transformation of o/p signal.

$G(s)$ = Forward path Transfor. function.

$H(s)$ = Feed back path Transfor. function.

$E(s)$ = Error Signal.

- Ex:
- i) Air conditioner.
 - ii) Automatic Iron.

ADVANTAGES OF OPEN LOOP CONTROL SYSTEM:

- It is very simple & easier to design.
- This type of system is very economical.
- This type of system is required less maintenance.

DIS-ADVANTAGES OF OPEN LOOP CONTROL SYSTEM:

- This type of control system is inaccurate.
- They are not reliable.
- This type of system are very slow processing.

ADVANTAGES OF CLOSE LOOP CONTROL SYSTEM:

- This type is more reliable.
- This type of system is very faster processing.
- In this type a no of variable can be handle sym-ultaneously.

DIS-ADVANTAGES OF CLOSE LOOP CONTROL SYSTEM:

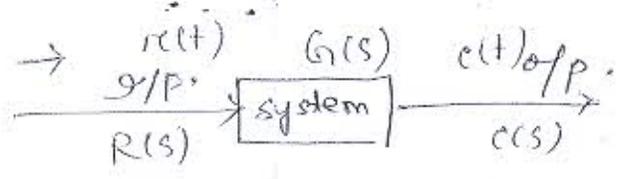
- This process are very expensive.
- This type of system having difficult-maintenance.
- This type is complicated ^{for} installation.
- They are less stable.

* **COMPARISON BET^N OPEN LOOP & CLOSE LOOP CONTROL SYSTEM.**

6

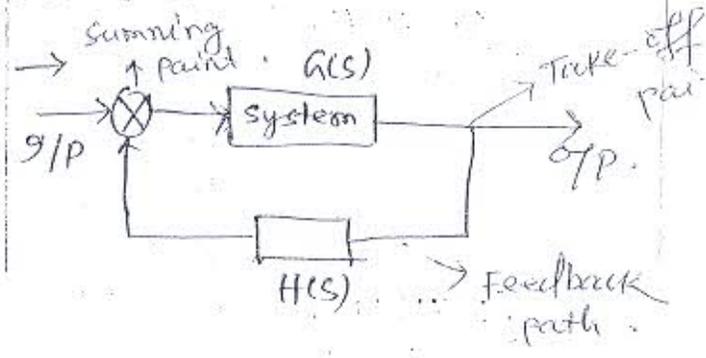
OPEN LOOP CONTROL SYSTEM

- It is otherwise called non-feedback type control system
- This type of system o/p is independent of the i/p.
- Feed back path are absent here.
- It is required less maintenance
- In this system are very slow processing.
- Ex:
 - i) Washing m/c.
 - ii) Traffic light system



CLOSE LOOP CONTROL SYSTEM

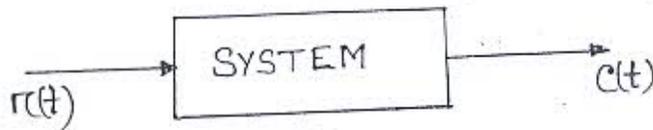
- It is otherwise called feedback type control system.
- This type of system o/p is dependent on the i/p
- Feed back path are present here.
- It is maintenance is so difficult
- This processing are very faster.
- Ex:
 - i) Air Conditioner.
 - ii) Automatic Iron.



MATHEMATICAL MODELING OF PHYSICAL SYSTEMS

TRANSFER FUNCTION:

Transfer function of a system is defined as the Ratio of Laplace transformation of o/p signal or variable to the Laplace transformation of i/p signal or variable, taking all initial condⁿ are "Zero".

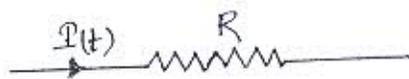


$$T.f = G(s) = \frac{C(s)}{R(s)}$$

PROCEDURE FOR DETERMINE A T.F OF SYSTEM:

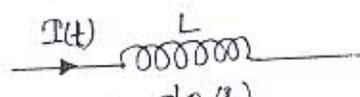
- Step: 1
Write down the integrodifferential equation for both i/p & o/p of the given system.
- Step: 2
Take the laplas transformation of above differential equation.
- Step: 3
Take the ratio of laplas transformation of o/p to the laplas transformation of i/p taking all initial condition are "Zero".
- Step: 4
The Ratio obtain in equation is the required Transfer function.

TRANSFER FUNCTION OF ELECTRICAL SYSTEM:



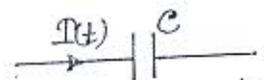
$$V_R = RI(t)$$

$$V_R(s) = RI(s)$$



$$V_L = L \frac{dI(t)}{dt}$$

$$V_L(s) = LS I(s)$$

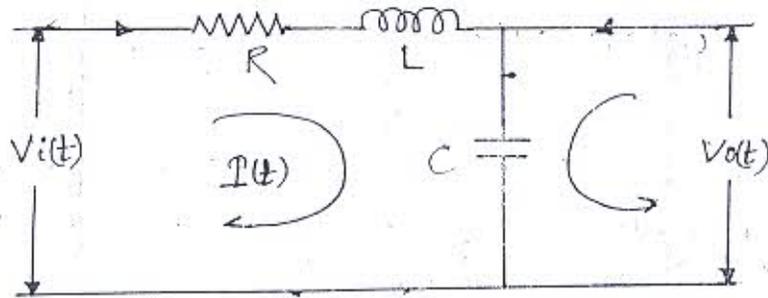


$$V_C = \frac{1}{C} \int I(t) \cdot dt$$

$$V_C(s) = \frac{1}{Cs} I(s)$$

⑧

* Determine The Transfer Function for The Given CKT:



$$V_o(t) = R I(t) + L \frac{dI(t)}{dt} + \frac{1}{C} \int I(t) dt \quad \text{--- (1)}$$

$$V_o(t) = \frac{1}{C} \int I(t) \cdot dt \quad \text{--- (2)}$$

Taking Laplace transformation of above equation.

$$V_i(s) = R I(s) + L s I(s) + \frac{I(s)}{Cs}$$

$$V_i(s) = I(s) \left(R + Ls + \frac{1}{Cs} \right) \quad \text{--- (3)}$$

$$V_o(s) = \frac{1}{Cs} I(s) \quad \text{--- (4)}$$

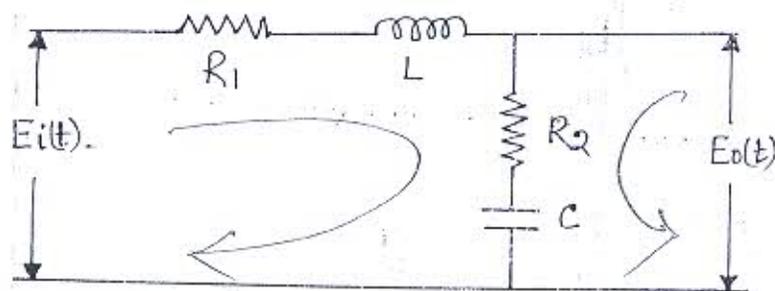
$$T.f = G(s) = \frac{V_o(s)}{V_i(s)}$$

$$= \frac{\frac{1}{Cs} I(s)}{\left(R + Ls + \frac{1}{Cs} \right) I(s)}$$

$$T.f = \frac{1}{Cs} \frac{I(s)}{R + Ls + \frac{1}{Cs}}$$

$$\Rightarrow T.f = \frac{1}{1 + Rcs + Lcs^2}$$

*



$$E_i(t) = R_1 I(t) + L \frac{dI(t)}{dt} + R_2 I(t) + \frac{1}{C} \int I(t) dt$$

$$\Rightarrow E_i(s) = R_1 I(s) + L s I(s) + R_2 I(s) + \frac{I(s)}{Cs} = I(s) \left[R_1 + R_2 + Ls \right]$$

$$E_o(t) = R_2 I(t) + \frac{1}{C} \int I(t) dt$$

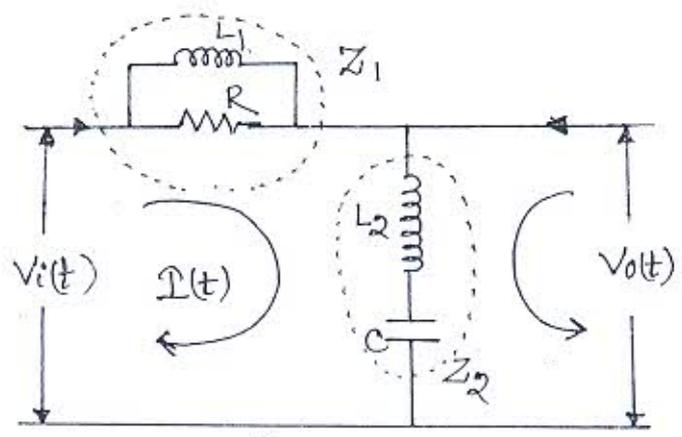
$$\Rightarrow E_o(s) = R_2 I(s) + \frac{I(s)}{Cs} = I(s) \left[R_2 + \frac{1}{Cs} \right]$$

9

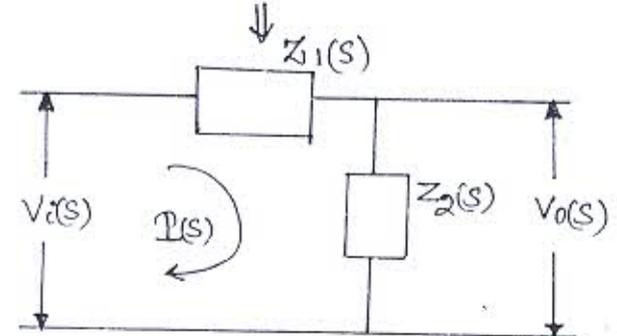
$$\begin{aligned}
 T.f = G(s) &= \frac{E_o(s)}{E_i(s)} \\
 &= \frac{R_2 I(s) + \frac{1}{Cs} I(s)}{R_1 I(s) + Ls I(s) + R_2 I(s) + \frac{1}{Cs} I(s)} \\
 &= \frac{I(s) [R_2 + \frac{1}{Cs}]}{I(s) [R_1 + R_2 + Ls + \frac{1}{Cs}]} \\
 &= \frac{R_2 Cs + 1}{R_1 Cs + R_2 Cs + Ls^2 + 1} \\
 &= \frac{1 + R_2 Cs}{Cs [R_1 + R_2] + Ls^2 + 1} \\
 \Rightarrow G(s) &= \frac{1 + R_2 Cs}{1 + Cs [R_1 + R_2] + Ls^2}
 \end{aligned}$$

$R \rightarrow R I(s)$
 $L \rightarrow L s I(s)$

*



$R \rightarrow R I(s)$
 $L \rightarrow L s I(s)$
 $L_2 \rightarrow L_2 s I(s)$
 $C = \frac{1}{Cs} I(s)$



$$\begin{aligned}
 Z_1(s) &= R \parallel L_1 s \\
 &= \frac{R L_1 s}{R + L_1 s} \\
 Z_2(s) &= L_2 s + \frac{1}{Cs} \\
 &= \frac{L_2 Cs^2 + 1}{Cs}
 \end{aligned}$$

KVL apply:

$$\begin{aligned}
 V_i(s) &= I(s) [Z_1(s) + Z_2(s)] \\
 &= I(s) \left[\left(\frac{R L_1 s}{R + L_1 s} \right) + L_2 \frac{Cs^2 + 1}{Cs} \right]
 \end{aligned}$$

$$\begin{aligned}
 V_o(s) &= I(s) Z_2(s) \\
 &= I(s) \left[\frac{L_2 Cs^2 + 1}{Cs} \right]
 \end{aligned}$$

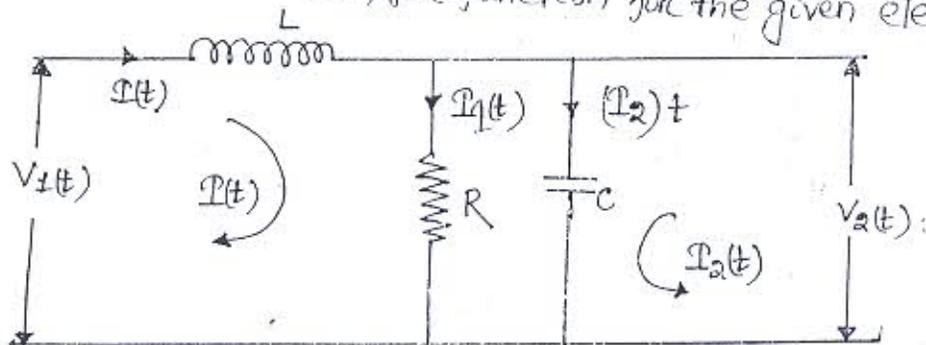
T.f = V_o(s)

10

$$= \frac{L_2 \frac{Cs^2 + 1}{Cs}}{\frac{RLs}{R+Ls} + \left(\frac{L_2 Cs^2 + 1}{Cs} \right)}$$

$$= \frac{L_2 \frac{Cs^2 + 1}{Cs}}{RLCs^2 + (L_2 Cs^2 + 1)(R+Ls)}$$

* Determine the transfer function for the given electrical N/w



Here, $(R || C) + L$

$$\text{So, } Z(s) = \left[R || \frac{1}{Cs} \right] + Ls$$

$$= \frac{R}{Cs} + Ls$$

$$= \frac{R}{Cs} + Ls$$

$$= \frac{R}{Rcs + 1} + Ls$$

$$= \frac{R + Ls + RLcs^2}{Rcs + 1}$$

$$V_1(s) = Z(s) I(s)$$

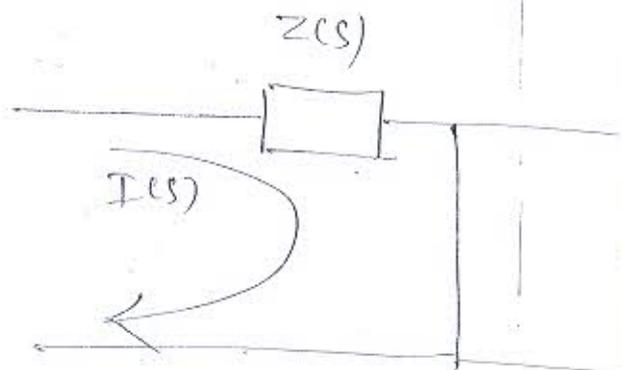
$$= \left(\frac{R + Ls + RLcs^2}{Rcs + 1} \right) I(s) \quad \text{--- (1)}$$

$$V_2(t) = \frac{1}{C} \int I_2(t) dt$$

$$V_2(s) = \frac{1}{Cs} I_2(s) \quad \text{--- (2)}$$

$$I_2(s) = \frac{I(s) \times R}{R + \frac{1}{Cs}} \quad \text{--- (3)}$$

$$= \frac{I(s)R}{Rcs + 1} = \frac{I(s)Rcs}{(Rcs + 1)}$$



Due to two different current $I(s)$ and $I_2(s)$, we have to find out the value of $I_2(s)$.

put eqnⁿ (3) in eqnⁿ (2)

$$\begin{aligned}
 V_2(s) &= \frac{1}{Cs} \times I_2(s) \\
 &= \frac{1}{Cs} \times \frac{Rcs}{(Rcs+1)} \times I(s) \\
 &= \frac{R}{(Rcs+1)} I(s) \quad \text{--- (2)}
 \end{aligned}$$

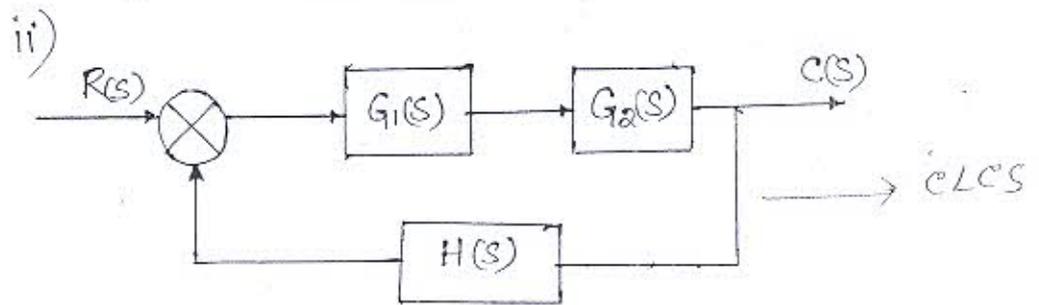
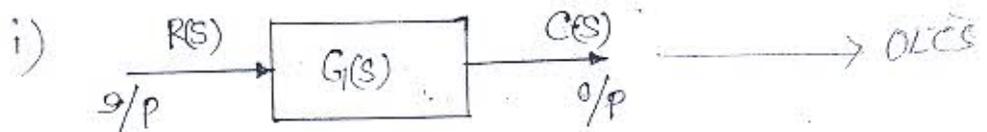
$$\begin{aligned}
 \therefore P.f &= \frac{V_2(s)}{V_1(s)} \\
 &= \frac{R}{\frac{R+Ls+RLCs^2}{(Rcs+1)}}
 \end{aligned}$$

$$T.f = \frac{R}{R+Ls+RLCs^2}$$

BLOCK DIAGRAM ALGEBRA:-

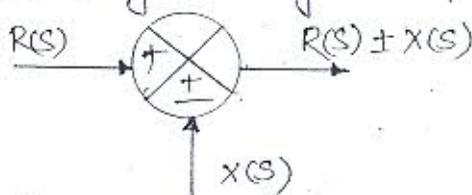
- Block diagram is the pictorial representation which is relating to input & out put of control system based upon the ~~cause~~ cause & effect approach.
- Block diag is the short hand representation of a T.F

Ex



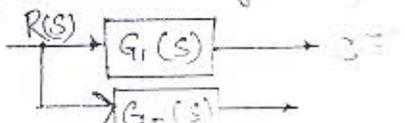
SUMMING POINT:

- Summing point represent summation of 2 or more i/p signal entering in a system.



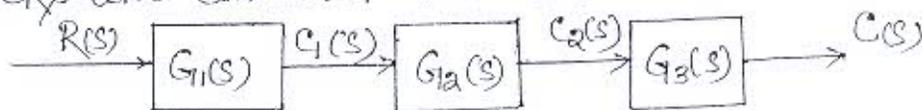
TAKE OFF POINT:

- Application of one i/p source to two or more system represented by a take off point.



BLOCK DIG. REDUCTION TECHNIQUE:

* Blocks are connected in cascade manner.



$$G(s) = \frac{C(s)}{R(s)}$$

$$C(s) = G(s) \cdot R(s)$$

$$C_1(s) = R(s) \cdot G_1(s)$$

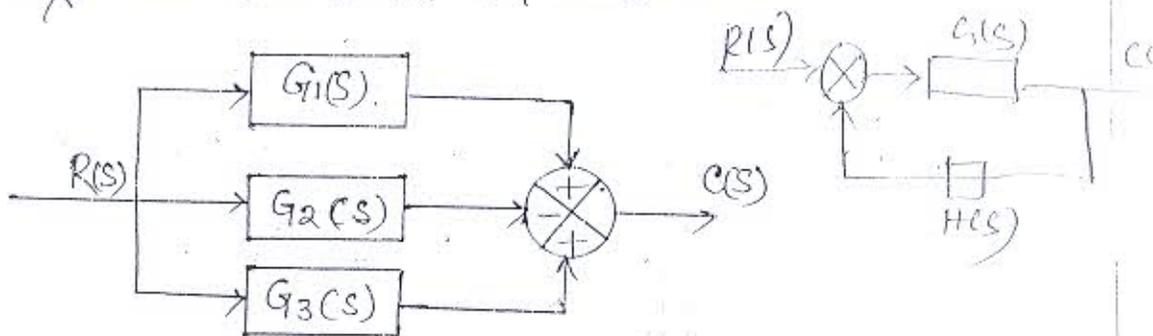
$$C_2(s) = C_1(s) \cdot G_2(s) = R(s) \cdot G_1(s) \cdot G_2(s)$$

$$C(s) = G_3(s) \cdot C_2(s)$$

$$C(s) = G_3(s) \cdot R(s) \cdot G_1(s) \cdot G_2(s)$$

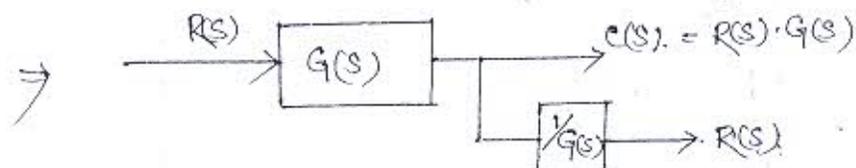
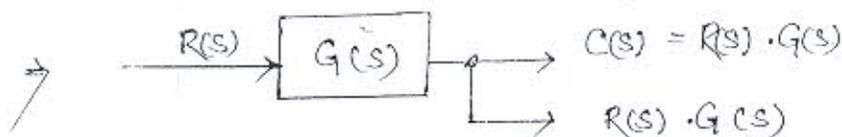
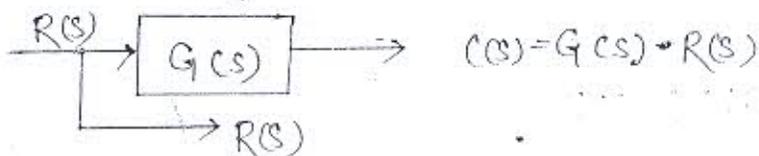
$$G(s) = \frac{C(s)}{R(s)} = G_1(s) \cdot G_2(s) \cdot G_3(s)$$

* Blocks are connected in parallel manner.



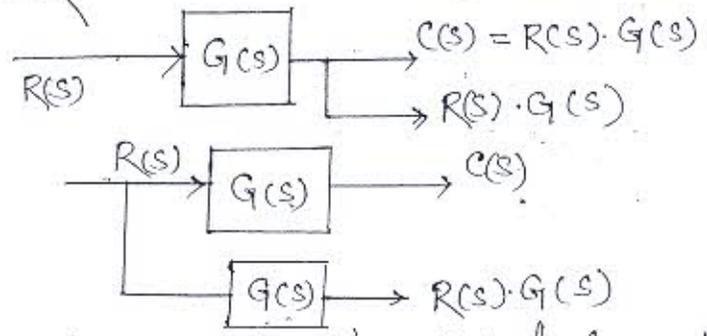
$$\frac{C(s)}{R(s)} = G_1(s) + G_2(s) + G_3(s)$$

* Shifting off take off point from before the block to after the block.

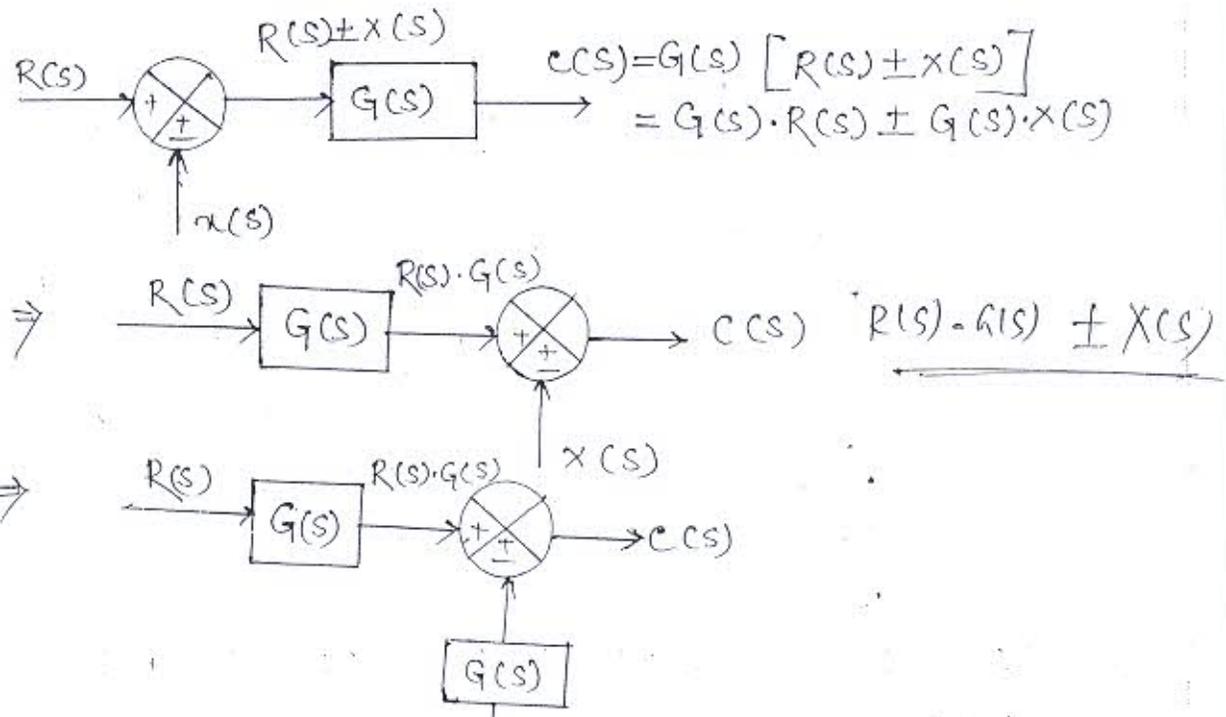


$$R(s) \cdot G(s) \times \frac{1}{G(s)} = R(s)$$

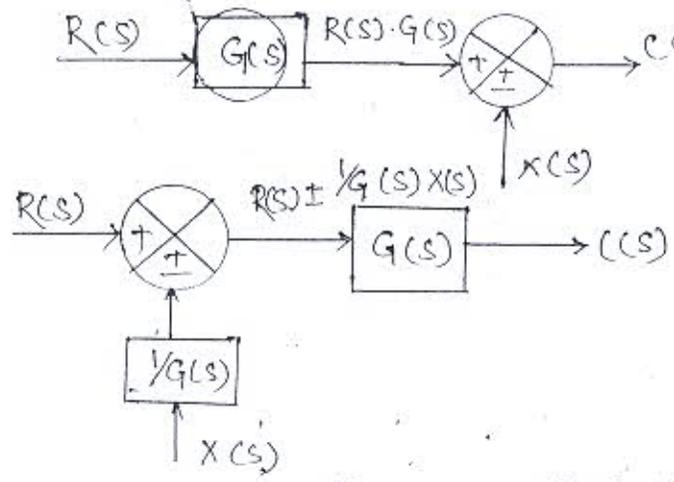
* Shifting the take off point from after the block to before the block.



* Shifting of summing point before the block to after the block.



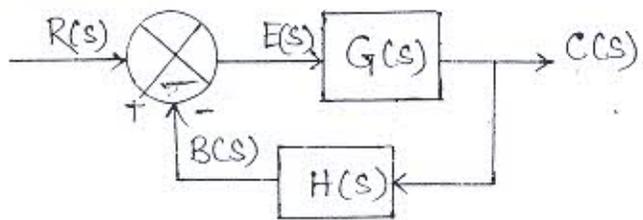
* Shifting the summing point after the block to before to block.



Take off point
 $L \rightarrow R \rightarrow \frac{1}{G(s)}$
 $R \rightarrow L = G(s)$
 Summing point
 $L \rightarrow R \rightarrow$
 $R \rightarrow L \rightarrow$

$\Rightarrow [R(s) \pm \frac{X(s)}{G(s)}] G(s) = C(s)$
 $\Rightarrow G(s)R(s) \pm X(s) = C(s)$

(14)

OVER ALL TRANSFER FUNC OF A CLOSED LOOP CONTROL SYSTEM

$$C(s) = G(s) \cdot E(s) \quad \text{--- (1)}$$

$$B(s) = C(s) \cdot H(s) \quad \text{--- (2)}$$

$$E(s) = R(s) - B(s) \\ = R(s) - C(s) \cdot H(s) \quad \text{--- (3)}$$

Putting eqⁿ (3) in eqⁿ (1) we get

$$C(s) = G(s) [R(s) - C(s)H(s)]$$

$$\Rightarrow C(s) = G(s) \cdot R(s) - G(s) \cdot C(s) \cdot H(s)$$

$$\Rightarrow C(s) + G(s) \cdot C(s) \cdot H(s) = G(s) \cdot R(s)$$

$$\Rightarrow C(s) \cdot (1 + G(s) \cdot H(s)) = G(s) \cdot R(s)$$

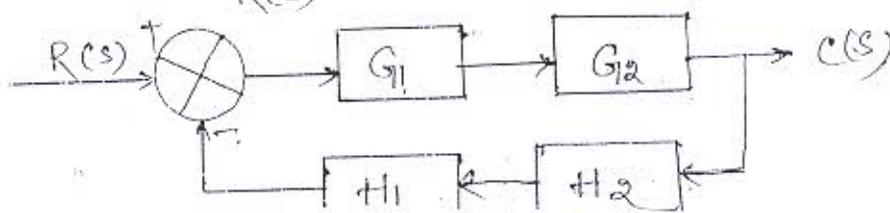
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{[1 + G(s) \cdot H(s)]} \quad \left(\text{For } \overset{-ve}{\text{feedback}} \text{ control system} \right)$$

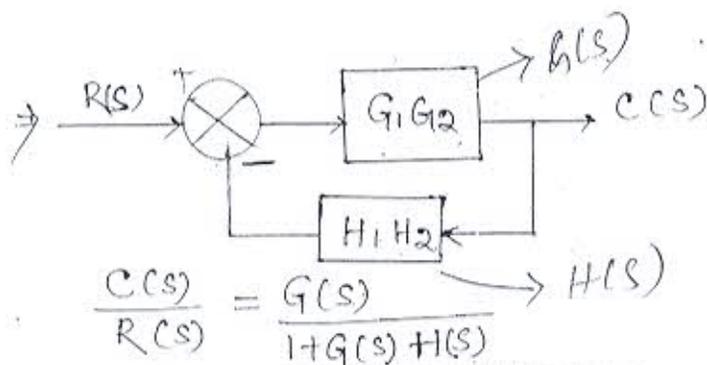
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{[1 - G(s) \cdot H(s)]} \quad \left(\text{For } +ve \text{ feedback control system} \right)$$

PROCEDURE FROM BLOCK DIAG. REDUCTION TECHNIQUE

- STEP-1: Reduced the block which are connected in cascade & series manner
- STEP-2: Reduced the block are connected in parallel
- STEP-3: Reduced the minor feed back loop
- STEP-4: Shift summing point to the left & take off to the right as far as possible.
- STEP-5: Repeat the steps from 1 to 4 till the canonical form is obtained.

✓ Det. the T.F $\frac{C(s)}{R(s)}$ for the given block diagram?

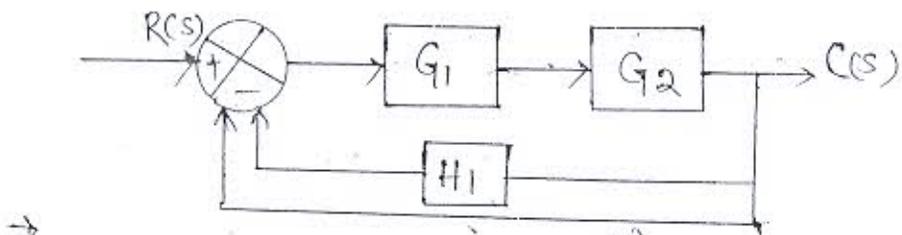




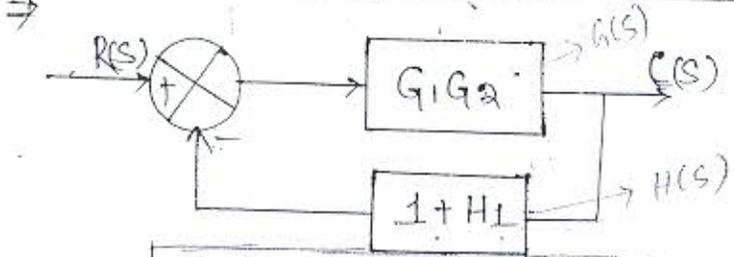
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1+(G_1G_2)(H_1H_2)}$$

Q. Det the T.F $\frac{C(s)}{R(s)}$ for the given block diagram.

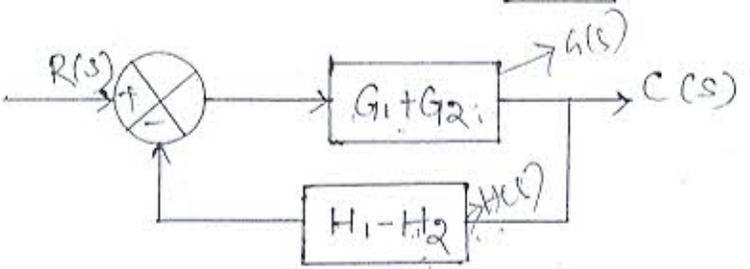
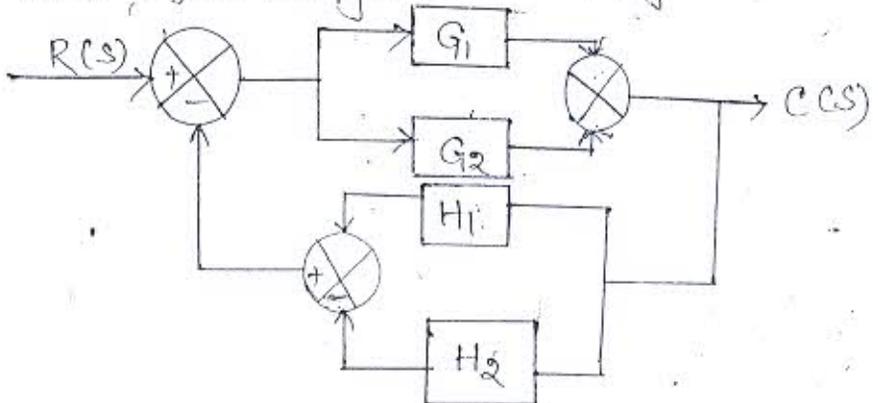


$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1+G_1G_2(1+H_1)}$$



$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1+G_1G_2(1+H_1)}$$

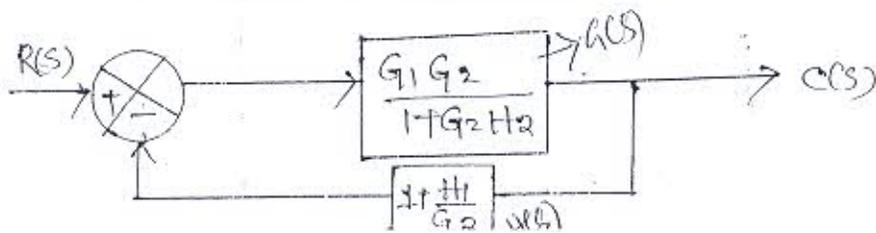
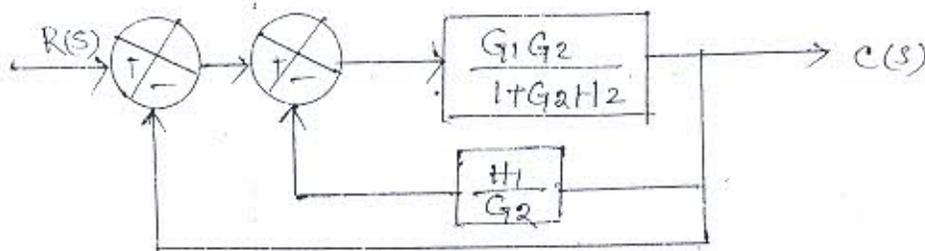
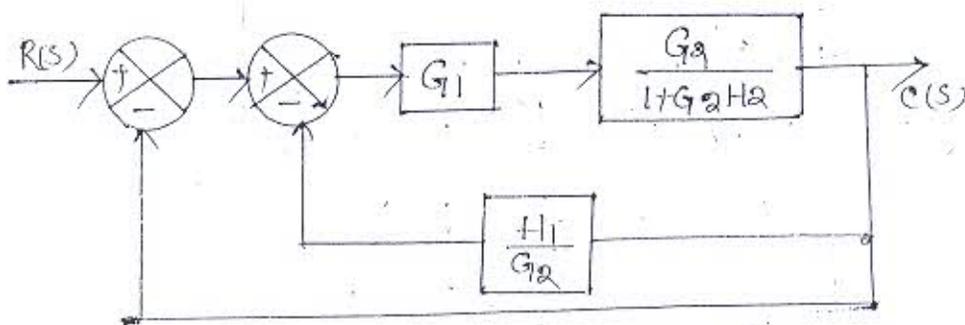
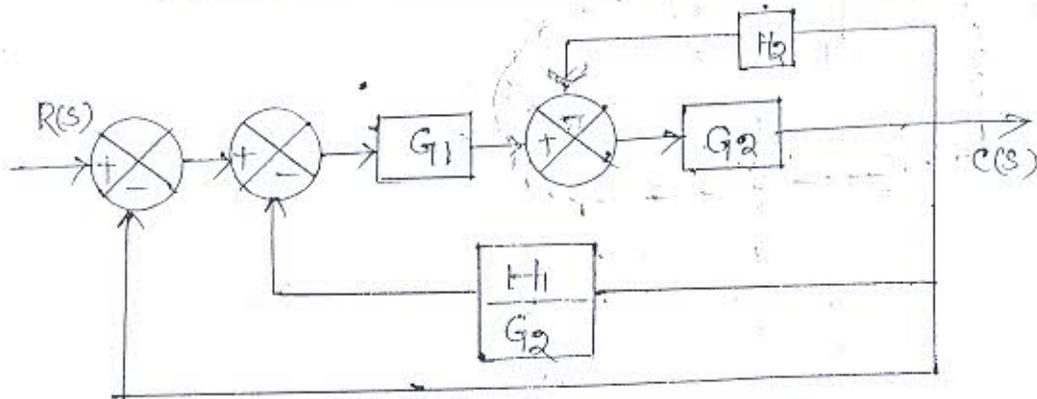
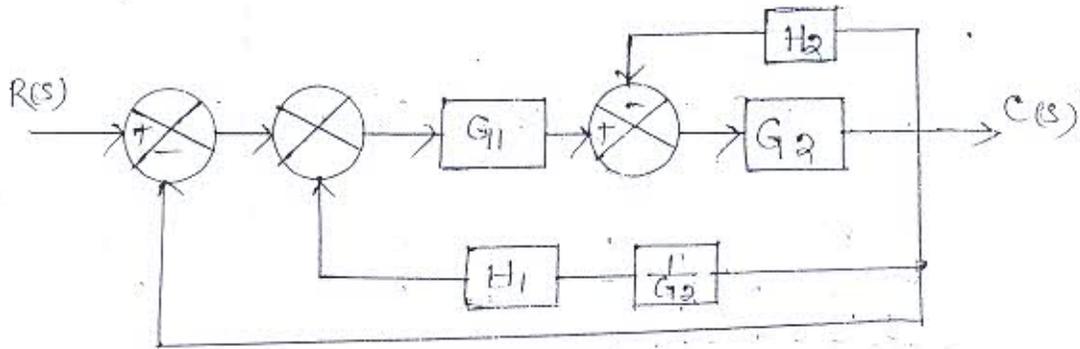
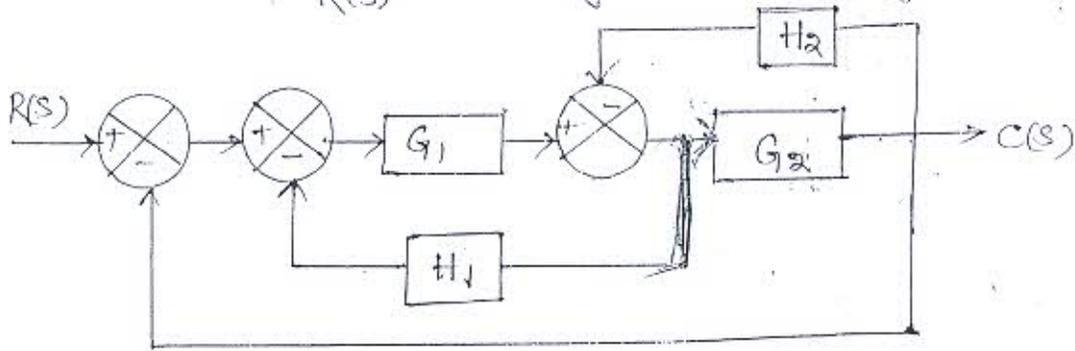
Q. Cal. the T.F for the given block dig.



$$\frac{C(s)}{R(s)} = \frac{G_1+G_2}{1+(H_1-H_2)(G_1+G_2)}$$

(16)

Q. Set the T.F $\frac{C(s)}{R(s)}$ for the given block dig:



$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 H_2}$$

$$1 + \left(\frac{G_1 G_2}{1 + G_2 H_2} \right) \left(1 + \frac{H_1}{G_2} \right)$$

$$= \frac{G_1 G_2}{1 + G_2 H_2}$$

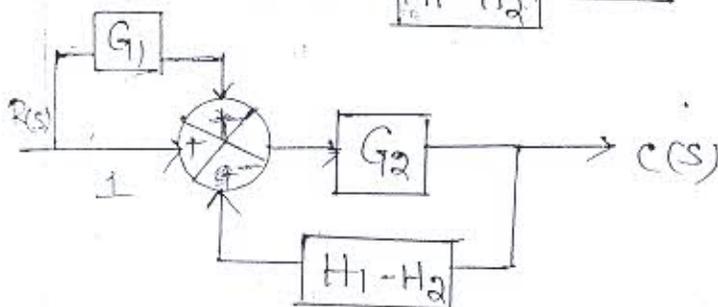
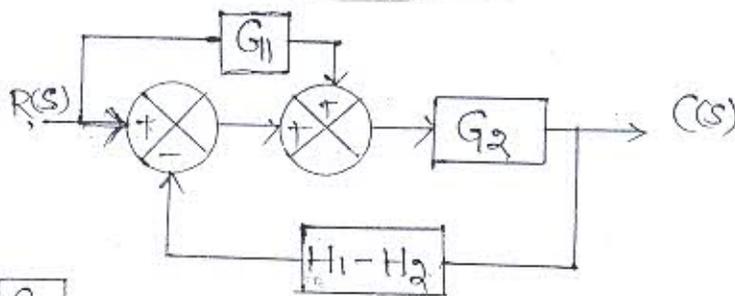
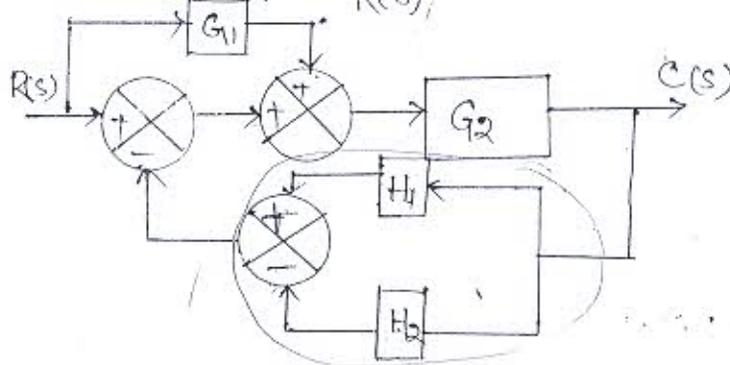
$$\frac{1 + \frac{G_1 G_2}{1 + G_2 H_2} + \frac{G_1 H_1}{1 + G_2 H_2}}{1 + G_2 H_2}$$

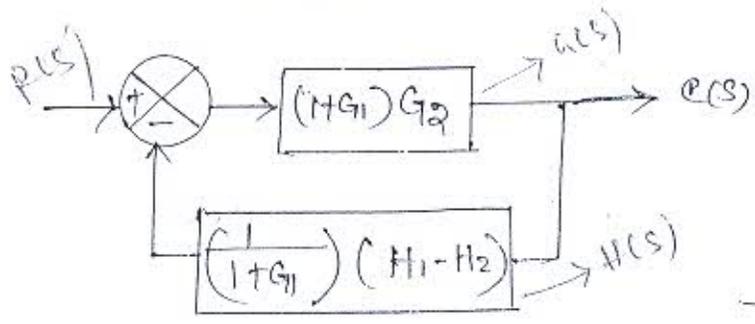
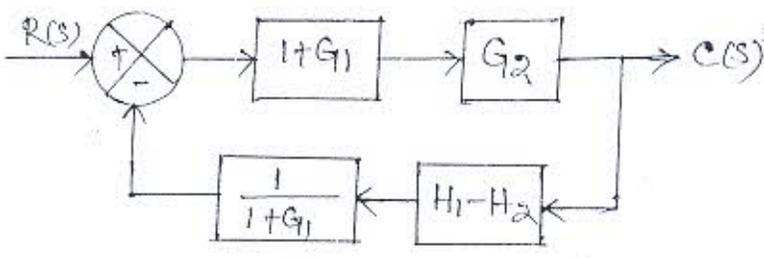
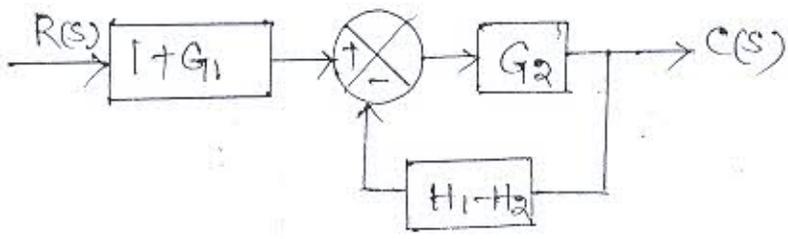
$$= \frac{G_1 G_2 / (1 + G_2 H_2)}{1 + G_2 H_2 + \frac{G_1 G_2}{1 + G_2 H_2} + \frac{G_1 H_1}{1 + G_2 H_2}}$$

$$(1 + G_2 H_2)$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 + G_1 H_1}$$

Q. Set the T.F. $\frac{C(s)}{R(s)}$ for the given block dig.





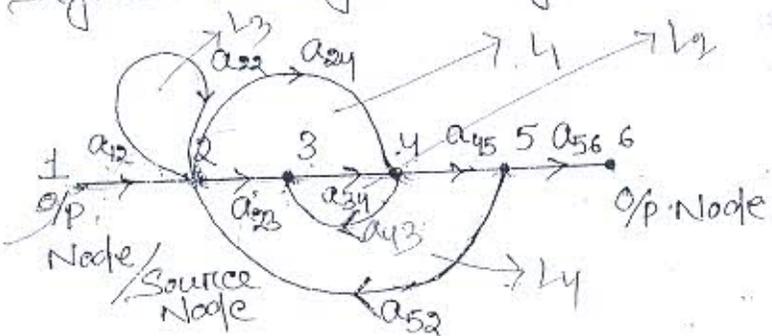
$$\frac{C(s)}{R(s)} = \frac{(1+G_1)G_2}{1 + \left(\frac{1}{1+G_1}\right)(H_1-H_2)(1+G_1)G_2}$$

$$H(s) = \frac{C(s)}{R(s)} = \frac{(1+G_1)G_2}{1 + (H_1-H_2)G_2}$$

(1)

SIGNAL FLOW GRAPH:

- A signal flow graph is a pictorial representation of simultaneous eqn describing a system.
- It graphically represents the transmission of signal through the system.



$a_{12}, a_{23}, a_{34}, a_{45}, a_{43}$ are known as transmittance function. Or transfer function value

MASON'S GAIN FORMULA:

- The relationship between an i/p variable and an o/p variable of a SFG is given by the net gain betⁿ the i/p & o/p nodes. This gain is called the overall gain of the system.
- In signal flow graph (SFG) method the transfer funcⁿ and the overall gain of a system can be calculated by Mason's gain formula.

It is given by →

$$T.F = \sum_{K=1}^n \frac{P_K \Delta_K}{\Delta}$$

Where,

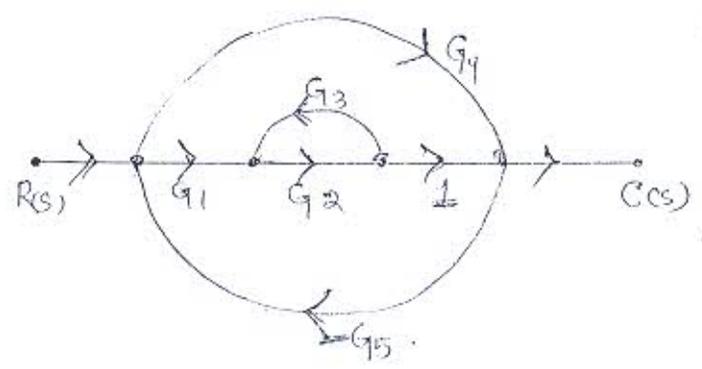
- P_K = gain of Kth forward path
- Δ_K = The part of the path p non-touching to Kth forward path.
- Δ = Determinant of the graph

$$\Rightarrow \Delta = [1 - (\text{Sum of all individual loop gain}) + (\text{Sum of all possible 2 non-touching loop gain}) + (\text{Sum of all possible 3 non-touching loop gain}) + \dots]$$

K = no. of Forward paths betⁿ i/p & o/p node.

∴ $\Delta_K = 1 - (\text{non-touching loop gains of forward path})$

Q.1 By using Mason's gain formula, det the T.F for the given signal flow graph.



Ans:

$$P_1 = G_1 G_2$$

$$P_2 = G_4$$

$$L_1 = -G_4 G_5$$

$$L_2 = G_2 G_3$$

$$L_3 = -G_1 G_2 G_5$$

2 Non-touching loop gain = $L_1 L_2 = -G_2 G_3 G_4 G_5$

$\Delta_1 = 1 - (0) = 1$

$\Delta_2 = 1 - (G_2 G_3) = 1 - G_2 G_3$

$$\Delta = 1 - (-G_4 G_5 + G_2 G_3 - G_1 G_2 G_5) + (-G_2 G_3 G_4 G_5)$$

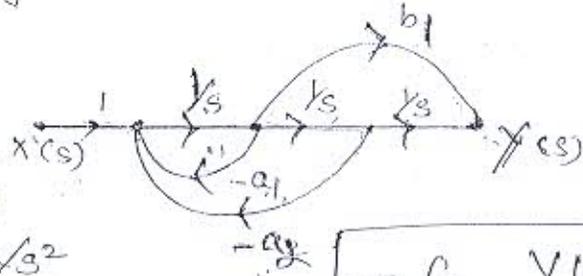
$$= 1 + G_4 G_5 - G_2 G_3 + G_1 G_2 G_5 - G_2 G_3 G_4 G_5$$

$$T \cdot F = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{(G_1 G_2)(1) + (G_4)(1 - G_2 G_3)}{1 + G_4 G_5 - G_2 G_3 + G_1 G_2 G_5 - G_2 G_3 G_4 G_5}$$

$$\Rightarrow T \cdot F = \frac{G_1 G_2 + G_4 - G_2 G_3 G_4}{1 + G_4 G_5 - G_2 G_3 + G_1 G_2 G_5 - G_2 G_3 G_4 G_5}$$

Q.3 Det the TF by using Mason's gain formula for the given SFG.



Q.3

$$P_1 = 1/s^3 \quad L_1 = -a_1/s$$

$$P_2 = b_1/s \quad L_2 = -a_2/s^2$$

$$\Delta_1 = 1 - 0 = 1$$

$$\Delta_2 = 1 - 0 = 1$$

$$\Delta = 1 - \left(-a_1/s + (-a_2/s^2) \right)$$

∵ Non-touching loop is absent.

$$\Rightarrow \Delta = 1 - (-a_1/s - a_2/s^2)$$

$$= 1 + a_1/s + a_2/s^2$$

$$\therefore T \cdot F = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{1/s^3 + b_1/s}{1 + a_1/s + a_2/s^2}$$

$$= \frac{1 + b s^2}{s^3}$$

$$\frac{s^2 + a_1 s + a_2}{s^2}$$

$$\Rightarrow T \cdot F = \frac{1 + b s^2}{s(s^2 + a_1 s + a_2)}$$

EFFECT OF FB ON OVERALL GAIN:

FOR OLCs $\Rightarrow T(s) = G(s) = \frac{C(s)}{R(s)}$

FOR CLCS $\Rightarrow T(s) = \frac{G(s)}{1+G(s) \cdot H(s)}$

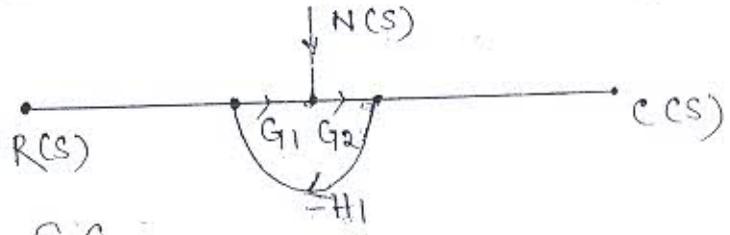
\therefore The overall gain of CLCS is reduced by the factor

$\left(\frac{1}{1+G(s) \cdot H(s)} \right)$

EFFECT OF STABILITY:

- \rightarrow In a closed loop system if $G(s)$ & $H(s) = 1$, then the system becomes unstable. Hence, the use of feedback (FB) may produce instability of operation.
- \rightarrow FB also makes an unstable system to a stable system.

EFFECT OF DISTURBANCE SIGNAL:



$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1+G_1 G_2 H_1}$

Here, $N(s)$ = disturbance signal

$\frac{C(s)}{N(s)} = \frac{G_2}{1+G_1 G_2 H_1}$ $R(s) \approx 0$

EFFECT OF SIGNAL DYNAMICS:

- \rightarrow In case of FB control system the system dynamics is better that means the transient response decrease rapidly.

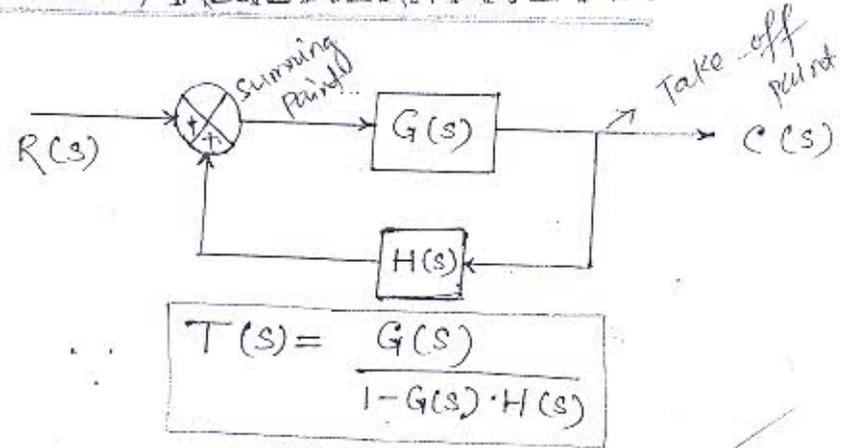
EFFECT OF BANDWIDTH:

- \rightarrow It is the range of freq. over which the system is operated.
- \rightarrow In case of -ve FB the Bandwidth increase (overall gain decrease). Whereas in case of +ve FB the B.W decrease (overall gain increase).

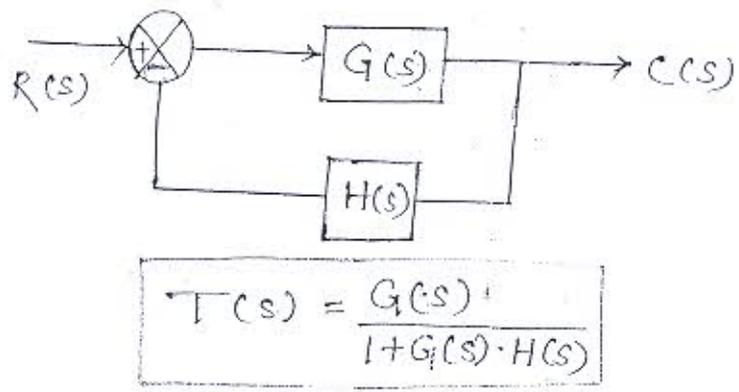
TYPES:

- \rightarrow Feedback are of 2 types \rightarrow
 - 1) +ve FB or regenerative FB
 - 2) -ve FB or degenerative FB

(1) +ve FB / REGENERATIVE FB:



(2) -ve FB / DEGENERATIVE FB:

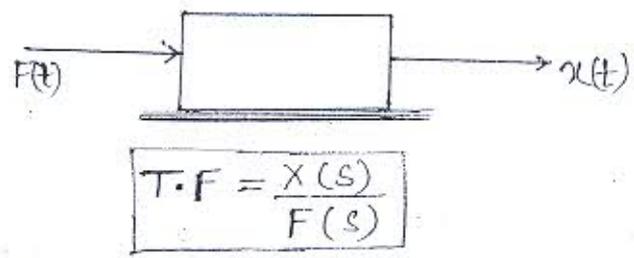


TRANSFER FUNⁿ FOR MECHANICAL SYSTEM:

→ In mechanical system there are 2 types of motion →
 i.e (i) Translational
 (ii) Rotational

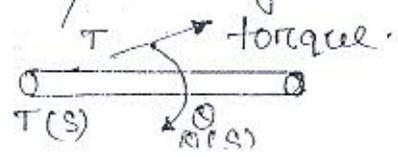
(i) TRANSLATIONAL MOTION:

→ In translational motion, the systems are characterized by linear displacement, linear velocity & linear acceleration.



(ii) ROTATIONAL MOTION:

→ Rotational mechanical systems are handled by the same as translational mechanical systems, except that the components undergo rotation instead of translation.



→ Here, torque replaces force and angular displacement replaces translational displacement.

$$T \cdot \theta = \frac{\theta(s)}{T(s)}$$

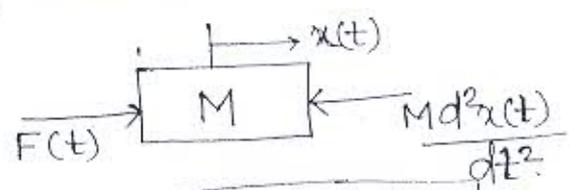
TRANSLATIONAL MOTION:

→ When a body moves along a straight line it is called so.
 → During the translational motion of body, it is opposing by 3 forces →

- (1) Inertia of the force.
- (2) Viscous force/viscous friction
- (3) Spring force

(1) INERTIA OF FORCE:

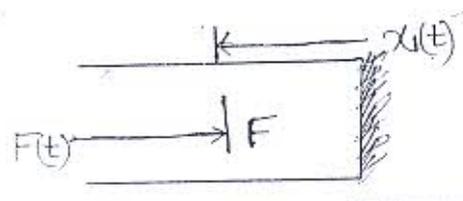
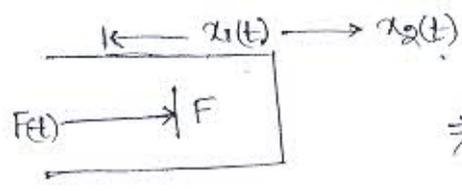
→ It is assumed that its mass is concentrated at its center. The function of mass in linear motion is to store kinetic energy. Mass can't store potential energy.



(2) VISCOUS FORCE / VISCOUS FRICTION:

→ When a force f is applied then the dashpot produces reaction, damping force f_B which is proportional to the velocity →

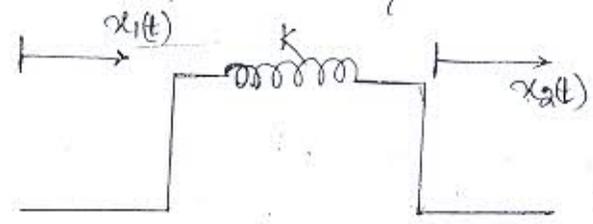
$$f_B \propto \frac{dx}{dt} \Rightarrow f_B = B \frac{dx}{dt}$$



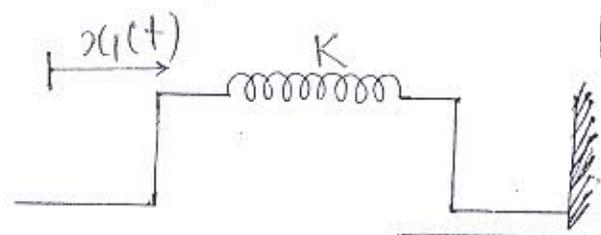
$$F_v(\dot{\phi}) = F \frac{d}{dt} x_1(t)$$

(3) SPRING FORCE :

→ If it is stretched, it tries to contract and if it is compressed, it tries to expand to its normal length.



$$F_s(t) = k[x_1(t) - x_2(t)]$$



$$F_s(t) = Kx_1(t) \quad (\because x_2(t) = 0)$$

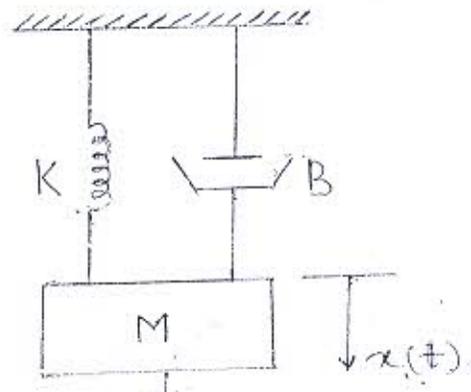
✓ D'ALEMBERT'S PRINCIPLE :

→ For any body the algebraic sum of externally applied forces and the force resistive motion in a given direction is zero.

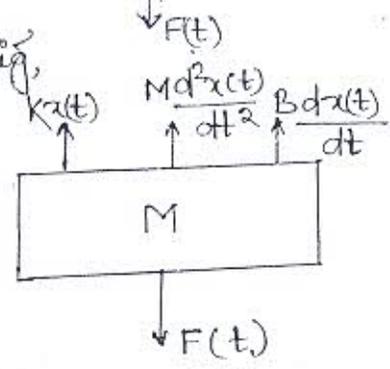
$$F(t) = F_M(t) + F_V(t) + F_S(t)$$

$$F(t) = M \frac{d^2x(t)}{dt^2} + F \frac{dx(t)}{dt} + Kx(t)$$

Q:1 Det. the T.F for the given Mechanical arrangement.



Free body dig.



$$F(t) = M \frac{d^2x(t)}{dt^2} + B \frac{dx(t)}{dt} + Kx(t)$$

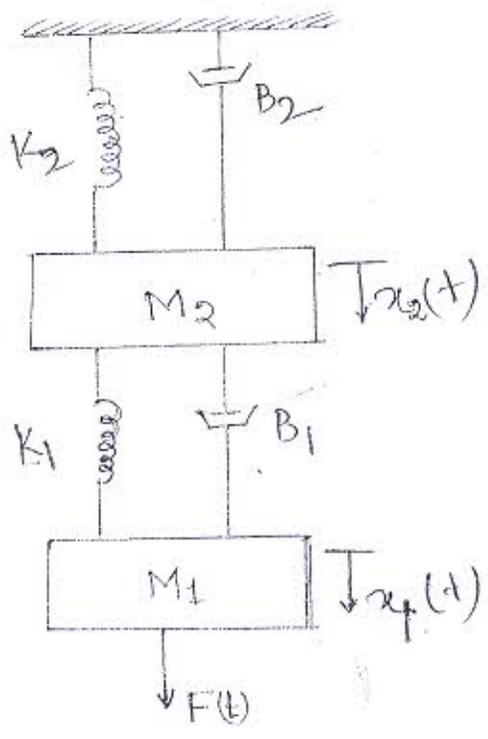
$$\Rightarrow F(s) = Ms^2x(s) + Bs x(s) + Kx(s)$$

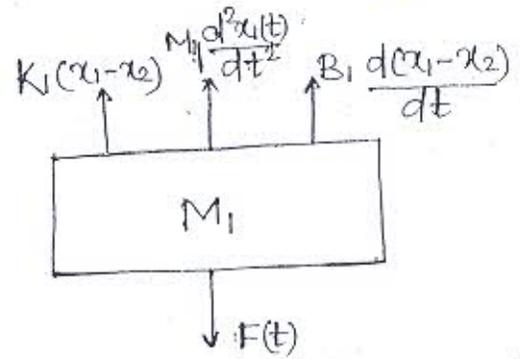
$$\Rightarrow F(s) = x(s) [Ms^2 + Bs + K]$$

① Taking Laplace Transformation eqn (1)

$$\Rightarrow \text{T.F.} = \frac{x(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

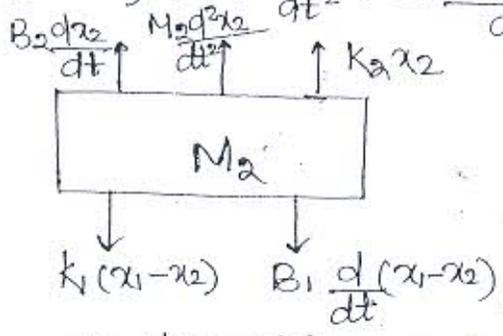
Q:2 Cal. the T.F for the given such arrangement.





$$\frac{x_2(s)}{F(s)} = ?$$

$$F(t) = K_1(x_1 - x_2) + M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{d(x_1 - x_2)}{dt} \quad \text{--- (1)}$$



$$K_1(x_1 - x_2) + B_1 \frac{d(x_1 - x_2)}{dt} = B_2 \frac{dx_2}{dt} + M_2 \frac{d^2x_2}{dt^2} + K_2 x_2 \quad \text{--- (2)}$$

eq(1) L.T $\Rightarrow F(s) = K_1 x_1(s) - K_1 x_2(s) + M_1 s^2 x_1(s) + B_1 s x_1(s) - B_1 s x_2(s)$

$$\Rightarrow F(s) = x_1(s) [K_1 + M_1 s^2 + B_1 s] - x_2(s) [K_1 + B_1 s] \quad \text{--- (3)}$$

eq(2) L.T $\Rightarrow K_1 x_1(s) - K_1 x_2(s) + B_1 s x_1(s) - B_1 s x_2(s) = B_2 s x_2(s) + M_2 s^2 x_2(s) + K_2 x_2(s)$

$$\Rightarrow K_1 x_1(s) + B_1 s x_1(s) = B_2 s x_2(s) + M_2 s^2 x_2(s) + K_2 x_2(s) + K_1 x_2(s) + B_1 s x_2(s)$$

$$\Rightarrow x_1(s) [K_1 + B_1 s] = x_2(s) [B_2 s + M_2 s^2 + K_2 + K_1 + B_1 s] \quad \text{--- (4)}$$

$$\Rightarrow x_1(s) = \frac{(B_2 s + M_2 s^2 + K_2 + K_1 + B_1 s)}{(K_1 + B_1 s)} x_2(s) \quad \text{--- (5)}$$

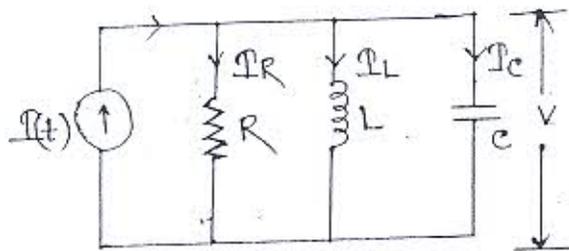
put eq(5) in eq(3)

$$\therefore F(s) = \frac{(B_2 s + M_2 s^2 + K_1 + K_2 + B_1 s) (K_1 + M_1 s^2 + B_1 s) x_2(s)}{(K_1 + B_1 s)}$$

$$\Rightarrow F(s) = x_2(s) \left[\frac{(B_2 s + M_2 s^2 + K_1 + K_2 + B_1 s) (K_1 + M_1 s^2 + B_1 s)}{K_1 + B_1 s} \right] \cdot \frac{1}{(K_1 + B_1 s)}$$

FORCE - CURRENT ANALOGY :

(27)



$$I = I_R + I_L + I_C$$

$$\therefore I(t) = \frac{V}{R} + \frac{1}{L} \int v dt + C \frac{dv}{dt}$$

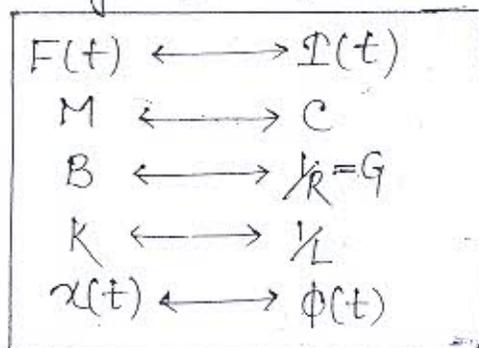
$$\Rightarrow I(t) = \frac{1}{R} \frac{d\phi}{dt} + \frac{\phi}{L} + C \frac{d^2\phi}{dt^2}$$

$$\Rightarrow I(t) = C \frac{d^2\phi}{dt^2} + \frac{1}{R} \frac{d\phi}{dt} + \frac{\phi}{L} \quad \text{--- (1)}$$

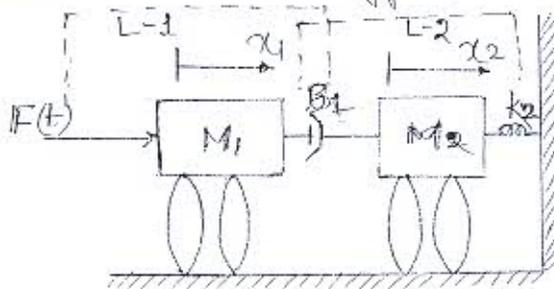
Similarly in mechanical system,

$$F(t) = M \frac{d^2x}{dt^2} + B \frac{dx}{dt} + kx \quad \text{--- (2)}$$

Comparing eqⁿ (1) & eqⁿ (2),

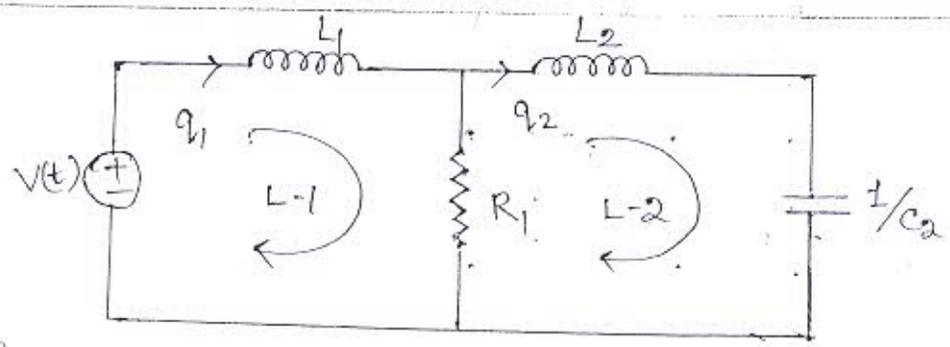


Q:1 Draw the F-V analogy of the given mechanical system?

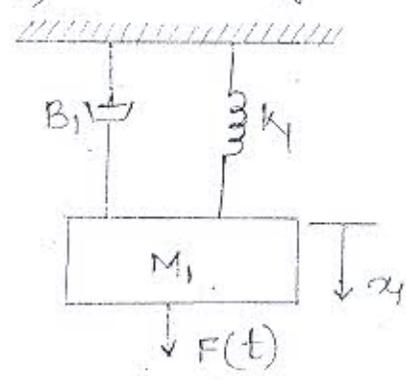


Procedure:

- In this case a 1st loop consists of a volt. source ($F(t)$), inductance ($L_1 \equiv M_1$), resistance ($R_1 \equiv B_1$), R_1 is common in betⁿ L_1 & L_2 loops.
- 2nd loop contains the inductance ($L_2 \equiv M_2$), resistance ($R_1 \equiv B_1$) and capacitance ($\frac{1}{C_2} \equiv k_2$)

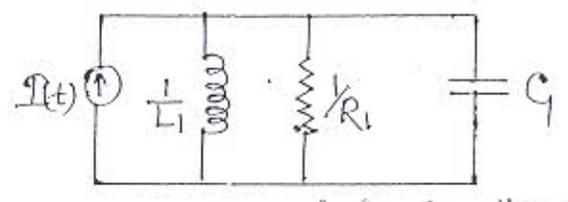


Q.2 Draw the force current analogy for the given mechanical system. (F-I)

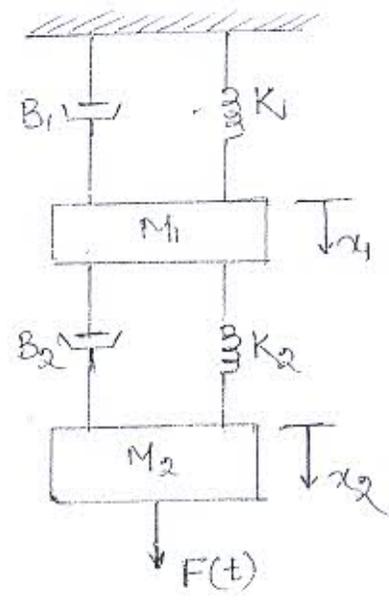


Q.3 Procedure:-

In this case $B_1 = R_1$ $M_1 = C_1$
 $K_1 = 1/L_1$ $x_1 = q_1(t)$
 $F(t) = I(t)$



Q.3 Draw the F-I analogy for the given mechanical system and also obtained a mathematical model for the given mechanical system.



Taking L.T of eqⁿ (1),
 $\Rightarrow MA(s) = Ms^2y(s) + Bsy(s) + Ky(s)$
 $\Rightarrow A(s) = \frac{y(s) [Ms^2 + Bs + K]}{M}$
 $= y(s) \left[s^2 + \left(\frac{B}{M}\right)s + \left(\frac{K}{M}\right) \right]$

Taking T.F is
 $\Rightarrow G(s) = \frac{y(s)}{A(s)} = \frac{1}{\left[s^2 + \left(\frac{B}{M}\right)s + \left(\frac{K}{M}\right) \right]} \quad \text{--- (2)}$

This displacement y is converted into volt. E_0 by using LVDT.

$E_0 \propto y$
 $\Rightarrow E_0(s) = Ky(s) \quad \text{--- (3)}$ $\frac{E_0(s)}{y(s)} = K_1$

From eqⁿ (2) & (3) we get,

$$G(s) = \frac{E_0(s)}{A(s)} = \frac{K_1}{s^2 + \left(\frac{B}{M}\right)s + \left(\frac{K}{M}\right)}$$

ROTATIONAL SYSTEM:

- In mechanical rotational system the force is replaced by torque, linear displacement is replaced by angular displacement, mass is replaced by moment of inertia & all other parameters are remain same.
- Here, torque is produced due to spring

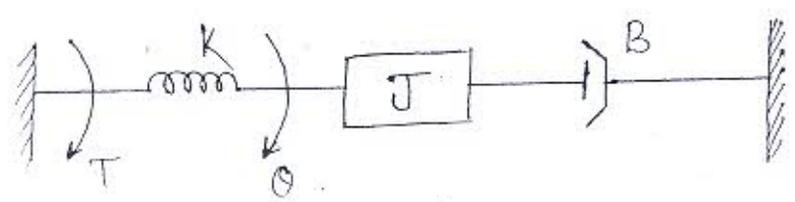
$$T_K = K\theta$$

- Torque due to moment of inertia.

$$T_i = J \frac{d^2\theta}{dt^2}$$

- Torque due to damping constant.

$$T_B = \frac{B d\theta}{dt}$$



Total torque $(T(t)) = T_K + T_i + T_B$
 $= K\theta + J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt}$

- $M \rightarrow J$
- $x \rightarrow \theta$
- $F \rightarrow T$

$$T(t) = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} + K\theta$$

→ Here θ is the function of time

Apply L.T. on both of eqⁿ (1)

$$\frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + Bs + K}$$

$$\Rightarrow \boxed{T(t) = J \frac{d^2 \theta}{dt^2} + B \frac{d\theta}{dt} + K\theta} \quad \text{--- (1)}$$

Here, θ = Func of time.

Apply L.T on the both side of eqⁿ (1) we get,

$$T(s) = Js^2\theta(s) + Bs\theta(s) + K\theta(s)$$

$$\Rightarrow T(s) = \theta(s) [Js^2 + Bs + K]$$

$$\Rightarrow \boxed{\frac{\theta(s)}{T(s)} = \frac{1}{(Js^2 + Bs + K)}}$$

FLUID SYSTEM:

→ Consider a pipe of length = l meter
diameter = D meter.

Viscosity of flow = μ

$U_{net} = N - \text{sec} / m^2$

Q = discharge through the pipe in cubic sec.

→ The pressure drop in pipe is given by,

$$P = \left(\frac{128 \mu l}{\pi D^4} \right) Q$$

$$\Rightarrow \boxed{P = RQ} \quad \text{--- (1)}$$

Where, $R = \frac{128 \mu l}{\pi D^4}$ = fluid of resistance.

→ The rate of fluid storage in the tank,

$$\boxed{dv = A \frac{dH}{dt}} \quad \text{--- (2)}$$

Where, A = Area of cross-section of pipe (m^2)

H = Height of the tank.

$$\boxed{P = \rho g H} \quad \text{--- (3)}$$

$$\Rightarrow \boxed{H = \frac{P}{\rho g}} \quad \text{--- (4)}$$

Put eqⁿ (4) in eqⁿ (2),

$$dv = A \frac{d}{dt} \left(\frac{P}{\rho g} \right)$$

Capacity of tank = $\frac{A}{\rho g}$

Control Hardware and their models.

(31)

Model-Based Design of CS.

- Simulation and testing with Model Based Design has become a best practice as motor CS continue to increase in complexity.
- Builders of complex equipment are driven to provide better performance while meeting tight deadlines and keeping costs down.
- One way to improve performance and lower costs is by improving motors and motor control systems.
- However, the demand for better performance and accuracy of motor control system is growing rapidly

and reaching the point where traditional design and verification methodologies are falling short.

- By using Model Based design, engineers can find errors earlier in the design process and create higher performing motor control systems.

—————XOX—————